

Chapter 13 Nuclear and Particle Physics

Overview

This, the last chapter, was intended to end this survey of progress from Galileo to “gluons.” (gluons are the exchange particles that bind quarks to form protons, neutrons and other hadrons). Unfortunately age has caught up with me and I only present the first section “Empirical Nuclear Information” to accompany and better explain the program CHARNUC. The end leaves us unsatisfied because it is not complete. Perhaps physics will never reach completion with new discoveries as long as there are humans. Although incomplete, the base of physical knowledge is so great that progress becomes increasingly difficult, as seen this century. Gone are the one man laboratories that were financed by the investigator. Progress today requires teams of investigators using massive apparatus requiring enormous budgets. Such a trend can hardly continue without limit.

This chapter continues the discussion started in Chapter 11 concerning the discoveries of the basic particles and the composition of nuclides. The size of the elementary particles indicates that our macroscopic experience provides no guide in this infinitesimal realm. The transition to the sub-atomic realm begins by organizing the nuclides in the Chart of the Nuclides that is provided on the distribution disk. This chart provides the measured neutral mass, lifetime, and statistics of the nuclides for guidance of theoretical models. A phenomenological explanation for the mass is provided by the semi-empirical mass formula. This leads to nuclear models such as the liquid drop which is the basis for the semi-empirical mass formula and for fission. The shell model is based on the fact that the nucleons in the nucleus are surprisingly independent in analogy to Bohr's model of the atom which provides insight into the “magic numbers” of neutrons and protons that are especially stable - similar to the magic numbers in the formation of atoms. Next we study how radiation interacts with matter and the type of radiation: alpha, beta and gamma and associated particles. Then we investigate the nature of the nuclear force that binds the nuclides into a nucleus. We should end by discussing the strange particles and models for their formation but alas time does not allow

Molecular Atomic and Particle Sizes

Our logical concepts are based on a world that can be seen by eye or seen through a microscope. In Chapter 8, wavelength was shown to determine the resolution of an optical microscope. For the sodium yellow line, the wavelength is 589 nm (nanometers - $1\text{E}-9\text{m}$). Figure 13.1-1 (top) indicates that molecules range in size from 10 to 1 nm or about 50 to 100 times too small to be seen optically.

Molecules, themselves, are formed of atoms each of which is about 0.1 nm diameter at the outer electron cloud

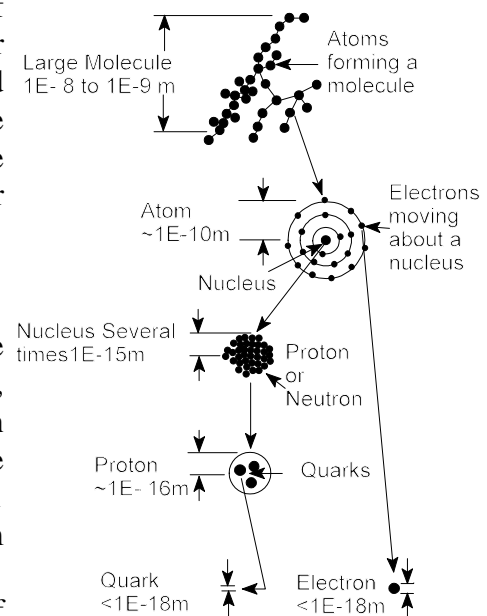


Fig. 13.1-1 The Size of Particles

about the nucleus. The nucleus itself is about one-thousand times smaller - several times 1 fm. (femto-meter - 10^{-15} m also called a Fermi). Nuclides are composed of protons and neutrons each of which are about 0.1 fm in diameter. A proton is thought to be composed of three *quarks*, two up quarks carrying $+2/3$ of an elemental charge and a down quark carrying $-1/3$ of the elemental charge to give $+1$ charge. Neutrons are thought to be composed of one up quark and two down quarks to give charge zero. Quarks and electrons have been shown to be less than 0.001 f hence maybe of no size. Thus to enter this nuclear world, we must jettison ideas of reality and intuitive judgment. Nature is our only guide.

13.1 Empirical Nuclear Information

13.1.1 Review

Chapter 11 introduced the primary particles that compose atoms or are associated with radioactivity. J.J. Thomson's discovery of the electron in 1897 was followed by Becquerel's discovery of radioactivity manifested as the emission of particles that α , β , and γ (α , β , γ in Greek). While Becquerel was investigating the radiation, the Curies were isolating the elements that produced the radiation and in so doing, discovered elements that result from uranium decay. Rutherford and Soddy showed that the emission of alpha and beta particles transforms radioactive isotopes into different chemical species. In 1912 Rutherford, Geiger and Marsden began probing the structure of atoms using alpha particles. These scattering experiments showed that the atom is not a "plum pudding" but a mostly empty structure with a small nucleus wherein resides practically all of the mass. By scattering from various elements, Rutherford discovered the proton - one of the fundamental nuclear constituents. These experiments revealed that the positive charge of the nucleus is equal to the atomic number used for chemically classifying the elements. Bohr (1913) working where these experiments were being done (University of Manchester) used the concept that the hydrogen atom is like a tiny solar system consisting of a proton and encircling electron. Using Planck's quantization, he predicted the line spectrum given by the Rydberg formula for the hydrogen atom.

But of what is the nucleus composed? It cannot be all protons because the atomic mass, A , is not equal to the atomic number, Z . It could be composed of A protons to get the observed mass, with $A-Z$ electrons to cancel $A-Z$ proton charges giving the correct charge. The β^- emission from some types of radioactive decay gives credence for electrons being in the nucleus. On the contrary, a nucleus composed of protons and electrons would have $2 \cdot A - Z$ particles instead of A . Spectroscopy showed that ^{14}C should have integral spin hence an even number of fermions in contradiction to the electron hypothesis. Furthermore the DeBroglie wavelength of electrons of momentum corresponding to the nuclear binding energy is too big to fit in a nucleus.

In 1931, Pauli postulated the existence of a mass-less, neutral particle with spin $1/2$, but this could not be a chargeless proton needed to explain the nuclear mass. In 1932, Chadwick discovered and named the neutron, a neutral particle with about the same mass as the proton. Pauli's particle was renamed "neutrino" by Fermi (Italian for little neutron).

Because the nucleus is so small, and because protons mutually repel each other, it is clear that the nuclear force (one of the four forces mentioned in Chapter 2) must be much stronger than the Coulomb force at short range but decrease with distance much more rapidly than the Coulomb force. A force with these properties accounts for the small size of the nucleus and the large amount of energy needed to remove a nucleon.

If a nucleus has no electrons, how is beta emission possible? Beta emission was explained by Fermi as the decay of a neutron into a proton, an electron and a neutrino. (The free neutron half-life is 14.8 minutes.)

In Chapter 12, Dirac's relativistic theory predicts that when an anti-particle regular particle annihilate their rest energies are released as gamma rays. The anti-particle of the electron is the positron. Since a neutron has slightly more mass than a proton, it would seem that a proton cannot decay into a neutron, but it can by emitting anti particles (anti-neutrino and anti-electron i.e. a positron).

Table 13.1-1 summarizes the particles known in the 1930s. These, I distinguish from the "strange" particles that were later produced at energies above several hundred MeV by accelerators, and found in cosmic rays. The top line lists the electron, (β^-) which was the first particle discovered. As mentioned, its anti-particle is the positron (β^+). Both have the same mass, but opposite elemental charges, they are called "leptons" (Greek for light weight). The electron is stable but it annihilates with a positron to release two γ rays each with energy $E = m_e * c^2$ where m_e is the rest mass of the electron. The third entry is not a particle but a convenient mass from which the others may be related: the atomic mass unit (AMU) which is the mass of carbon 12 divided by 12. It is also

Name	Mass (kg, MeV, AMU)	Charge	Type	Stability	Discoverer
electron, β^- , beta minus	0.91095E-30 0.51099 5.4858E-4	negative	lepton	stable	J.J. Thompson, 1897
positron, β^+ , beta plus	same as electron	positive	lepton	annihilates with an electron	Anderson, 1932
AMU	1.665655E-27 931.5016 -				
proton, p	1.672648E-27 938.28 1.007276	positive	baryon	stable	Rutherford, 1914
neutron, n	1.674854E-27 939.573 1.008665	none	baryon	when free decays in 888 s	Chadwick, 1932
gamma, γ , photon	none	none	boson	stable	Becquerel, 1896
neutrino, ν_e , electron neutrino	none	none	lepton	>300s	Pauli predicted, 1933, Cowan & Reines,
deuteron, d	3.343637E-27 1875.628 2.013553	positive		stable	Urey, 1932
triton, t	5.0288E-27 2808.9 3.0155	positive		12.3y decays to $\text{He}^3 + \beta^-$	
alpha, α	6.644763E-27 3727.409 4.001506	two positive		stable	Rutherford, 1903

expressed in MeV using Einstein's mass energy relationship. The fourth entry is the proton which is the nucleus of hydrogen; it is followed by the neutron, the neutral duo of the proton found in nuclei (except hydrogen). The sixth entry is the charge-less, mass-less particle of electromagnetic radiation. It is the particle from nuclear de-excitation. The seventh particle is the charge-less and believed to be mass-less, neutrino that accompanies beta decay and like an electron is a lepton. The eighth entry is the deuteron which is the nucleus of deuterium containing one proton and one neutron. The triton (9th entry) is the nucleus of tritium containing two neutrons and one proton. The last entry is the alpha particle that Becquerel observed from radioactive decay consisting of two protons and two neutrons.

Nomenclature

Our notation for a neutral isotopic species i.e. a nucleon with its neutralizing cloud of electrons, is: ${}^Z\text{Chemical Symbol}^A$, where Z is the number of protons causing the nuclear charge; it is called the atomic number; A is the mass number which is the sum of protons and neutrons: $A = Z + N$, with N being the number of neutrons. For example, ${}^1\text{H}^1$ is the common form of hydrogen. This nomenclature is redundant because the chemical symbol, H, implies $Z = 1$ hence the left superscript may be suppressed: ${}^1\text{H}^1 \rightarrow \text{H}^1$.

A nuclear reaction may be written like a chemical equation. For example, neutron capture in hydrogen forms deuterium with excess energy being emitted as a gamma particle. This reaction is written: $n + {}^1\text{H} \rightarrow {}^2\text{H} + \gamma$. Charge and mass-energy are conserved.

Basic Facts about Nuclei

The nuclear charge Z determines all of the chemical properties of an element. Z ranges from 0 to 103 according to Table 11.11-1. The mass ranges from $A = 1$ to 255 with a few not found in nature. Missing are $A = 5$ and 8 which are so short lived that they cannot even be produced in the laboratory. Nuclei having the form $A = 4*n+1$ beyond $A = 209$ are not found in nature but many have been artificially produced. Table 13.1-2 lists words prefixed with "iso" that show nuclear similarities.

Table 13.1-2 Isos	
Isotope	Nuclides with the same number of protons, Z
Isobar	Nuclides with the same number of nucleons, A
Isotone	Nuclides with the same number of neutrons, N
Isomer	Nuclides having the same A and Z but different decay times due to the need for multipole transitions

Isotopes

Nuclei having the same Z (the same chemical properties), but different A s (mass numbers) are isotopes (same place in the Periodic Table). On the average, elements have about three isotopes. For example, the three stable isotopes of silicon are Si^{28} , Si^{29} and Si^{30} . Additionally, most elements have radioactive isotopes which for silicon are Si^{27} and Si^{31} . The former of which decays with a half-life of 4 s according to $\text{Si}^{27} \rightarrow \beta^+ + \text{Al}^{27}$ because it has too much charge for its mass. The latter of

which decays with a half-life of 170 m according to $Si^{31} \rightarrow \beta^- + P^{31}$ because it has too much mass for its charge.

Isobars

Isobars (same weight) are nuclides with the same number of nucleons, A , but differ in Z or N . Examples of isobars are H^3 and He^3 (tritium and helium-3). Also S^{36} and A^{36} , Ru^{104} and Pd^{104} as well as the triplet Zr^{96} , Mo^{96} and Ru^{96} .

Isotones

Isotones (same neutrons) are nuclides with the same N , but differ in their numbers of protons. For example, helium-4 and tritium (He^4 and H^3) are isotones because have the same number of neutrons (2), although their Z is 2 and 1, respectively.

Isomers

Isomers (same part) are nuclides in which the lowest excited state has a sufficiently different spin from the ground state causing a long half-life, the excited state is called metastable or an isomer of the nucleus. The excited isomer is usually designated by an "*": In^* was the first one observed. Nuclei with this property are identified in the Chart of the Nuclides.

Regularities

The stable nuclei show striking regularities. Nuclei of even Z or A are much more numerous than those having odd Z or A . Nearly all nuclei with even A have even Z except: H^2 , Li^6 , B^{10} , and N^{14} . This fact is what makes stable nuclei with even Z more numerous than those with odd Z , for a nucleus with even Z may have either odd or even A . These three rules are illustrated in Table 13.1-3.

Z	# of Stable Isotopes	# with odd A	# in even A
48	8	2	6
49	2	2	0
50	10	3	7
51	2	2	0

13.1.2 Chart of the Nuclides (the Segré Chart)

The chart of the nuclides is a plot of Z vs N (Z is the ordinate) of the known isotopes¹ This book provides a condensed version on the distribution disk entitled CHARNUC that uses data from

¹ The chart is available from the General Electric as a 30 x 53.5 in. wall chart from General Electric Co., Nuclear Operations, 175 Curtner Ave. M/C684, San Jose CA 95125, or from Karlsruhe (KfK) Laboratory from Gersbach u. Verlag, Isoldenstr, 38, D-8000, München 40. It is also available from the Lund Nuclear Data WWW Service at web site: <http://nucleardata.nuclear.lu.se/nucleardata/>.

the National Nuclear Data Center at Brookhaven National Laboratory (<http://www.nndc.bnl.gov/>). It can be run from the distribution disk but more conveniently by copying the disk to your hard drive. When done, open the folder CHARNUC and double click on the "bat" file (slightly different procedures are needed if using DOS). This presents a banner and brief instructions. Pressing the space bar results in instructions to input the chemical symbol for your element of interest followed by the mass of the isotope.

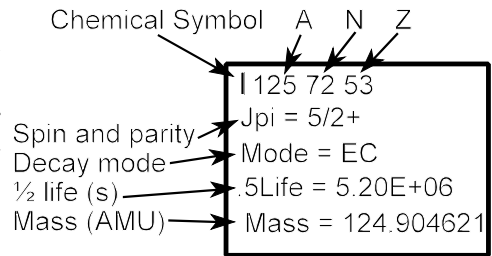


Fig. 13.1-2 Isotopic Data

For example, typing I125 results in an information box for the isotope ^{125}I (Figure 13.1-2) centered on the screen with its 24 neighbors. If you don't know the mass, type X*, where X is the chemical symbol of interest.

Figure 13.1-2, first line, shows the chemical symbol, the mass number, the number of neutrons and number of protons. The next line gives the isotopic spin and parity, the following line gives the types of decay. (This example decays by electron capture with a half-life of $5.20\text{E}+6$ s and has a nuclear mass of 124.904621 AMU.) The reason why nuclear masses are not integer is that the binding energy causes the mass to change according to: $E_{\text{binding}} = [M(A,Z) - A] * c^2$, where $M(A,Z)$ is the measured mass of a nuclide with these A and Z.

Note: mass given in the Chart of the Nuclides is the mass of the neutral atom i.e. it includes the electrons. Generally allowance for the electrons is made by using the mass of hydrogen in place of the proton mass. The exceptions to this practice are calculations involving β^+ decay when the electron masses must be included explicitly.

When in the screen showing 25 isotopes, pressing "w" (where) shows the location of the isotope at which you were just looking with respect to all of the others isotopes (Figure 13.1-3 with the position indicating lines deleted to avoid confusion). The N, Z coordinates of more than 2000 nuclides are displayed. Some elements have many isotopes, but the lightest elements have only a few. For example tin, Sn, has 23 isotopes ranging from ^{108}Sn to ^{132}Sn (some are missing) while hydrogen only has three isotopes. Tin is "magic" in having 50 protons with one isotope magic in 126 neutrons.

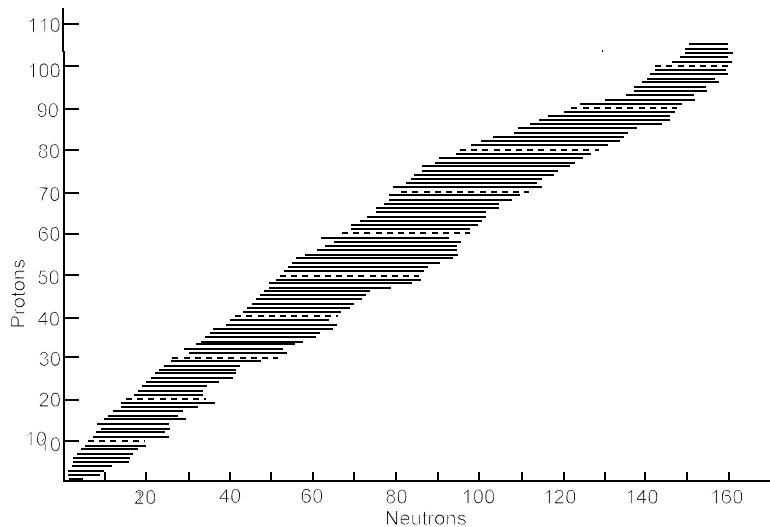


Fig. 13.1-3 Overall Perspective of the Nuclides

(Dotted lines indicate a multiple of 10)

Ratio of Neutrons to Protons

The nuclear force is the same between protons and neutrons as it is between protons and protons as it is between neutrons and neutrons.. The ratio of neutrons to protons is about 1 for the light elements but neutrons dominate as the nuclear mass increases. This is because protons are attracted by the nuclear and repelled by the Coulomb force which increasingly repels them as their number increases. To investigate this relationship, Computer Program 8-3 Least-Squares Fitting of the Cauchy Coefficients was rewritten to fit Z vs N and A vs N , in separate modifications of the program. (The modified program is included as Computer Program 14-1, but not discussed.) The major changes to the program were the replacement of the optical data with 98 values of Z and the atomic mass for each element. These data were taken from the *Periodic Chart of the Elements* (Table 11.11-3). An average N value was estimated by subtracting Z from the atomic mass. The fitted results are shown in equations 13.1-1 to 13.1-6 with the first and last being power-law fits. The ratio: $Z/N = 0.646$ results because of the many more heavy than light nuclides in the table. In the region of light nuclides the ratio of $Z/N = 1$. The same computer program finds the ratio: $A/N = 0.746$. Equations 13.1-2 to 13.1-5 are polynomial fits. The first order fits (equations 13.1-2 and 13.1-4) are obviously poor (seen by substituting $N = 1$), however the second order fit predicts the approximately correct Z and has a slope of 0.909 for small N .

$$\begin{aligned}
 Z &= 3.15 * N^{0.646} && 13.1-1 \\
 Z &= 23.04 + 0.37 * N && 13.1-2 \\
 Z &= 0.346 + 0.909 * N - 0.002 * N^2 && 13.1-3 \\
 A &= 23.04 + 1.37 * N && 13.1-4 \\
 A &= 0.347 + 1.91 * N + 0.002 * N^2 && 13.1-5 \\
 A &= 4.95 * N^{0.746} && 13.1-6
 \end{aligned}$$

13.1.2 Nuclear Reactions

Spontaneous or Absorption Reactions

The chart of the nuclides conveniently shows the transformation of a nucleus when it absorbs or emits a particle. Figure 13.1-4 shows the location of the transformed nucleus relative to the original nucleus in a plot of Z vs N with the origin (0,0) at the original nucleus. A shift to the left results in a new nucleus with less neutrons than the original; a shift to the right results in a new nucleus with more neutrons than the original. A shift up results in a nucleus with more positive charges, a shift down results in a nucleus with less positive charge than the original.

As examples, if the original nucleus emits an alpha particle, the nucleus loses two protons ($Z-2$) and two neutrons ($N-2$) thus: ${}^A\text{X}^N \rightarrow {}^{A-4}\text{X}^{N-2} + \alpha$. In electron conversion, ϵ , the original nucleus effectively absorbs an

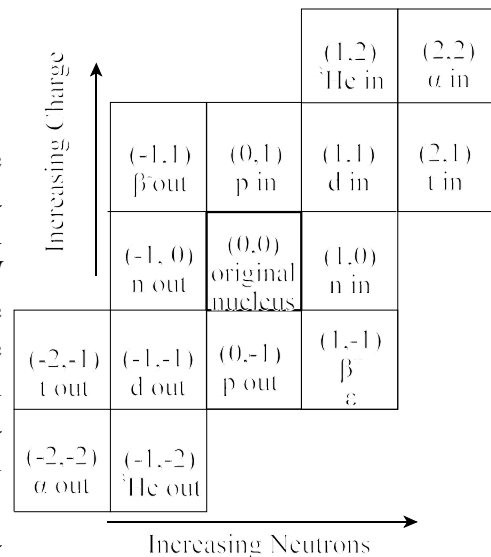


Fig. 13.1-4 Emission or Absorption of a Particle.

electron (usually from the *K* orbit thus canceling a positive charge². This reduces the charge by 1 and increases the number of neutrons is one [Figure 13.1-4 location (1,-1)].

Particle bombardment

Nuclides are transformed by particle bombardment, symbolized as "particle in, particle(s) out". For example, n,p scattering consists of a neutron incident on a nucleus and a proton coming out. Since the chart of the nuclides is a plot of *N* vs *Z*, a neutron-in shifts right 1 space; a proton-out shifts down one space to produce a new nucleus at location (1,-1) if the original nucleus was at (0,0). Similarly, an n,α reaction removes 1 neutron and adds 2 protons to result in a new isotope at location (1,2). Figure 13.1-5 uses the Segré chart to show how the original nucleus is changed by the reactions represented by the symbols: n - neutron d - deuteron, p - proton ³He helium-3, t - triton (³H) α alpha particle, γ - gamma ray.

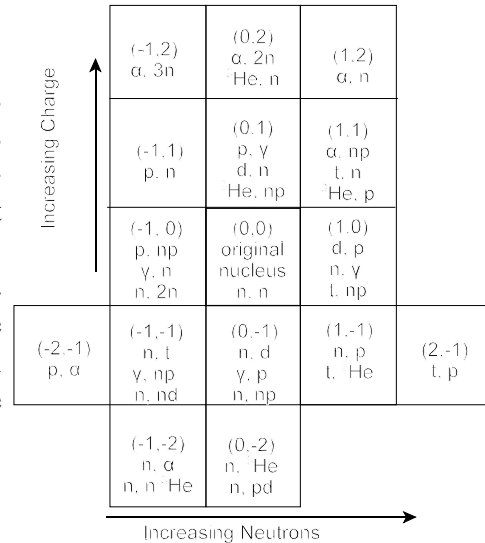


Fig. 13.1-5 Nuclear Reactions

Summary of the Chart of the Nuclides

It is clear that whether or not a reaction is spontaneous or due to incident and exiting particles, charge and neutron number are conserved³. Charge conservation is expressed in equation 13.1-7; neutron conservation by equation 13.1-8 where *R* represents the resulting nuclides, *T* for the target nuclides, *i* for the incident particle, and *o* for the out particle.

13.1.3 Binding Energy of Nuclides

Mass Loss

A nucleus is held together by strong, short-range, nuclear forces that overcome the Coulomb repulsion of the protons. The mass of a nucleus is less than the mass of all of the protons and neutrons that it contains. This mass reduction occurs because the missing mass is converted into the energy to bind

$$Z_R = Z_i - Z_o + Z_T \quad 13.1-7$$

$$N_R = N_i - N_o + N_T \quad 12.1-8$$

in the absence of β processes

$$BE(MeV) = [Z * m_H + (A - Z) * m_n - atwt(Z, N)] * 931.5016 \quad 13.1-9$$

² The reaction is: $\nu + \beta^- + \underline{p} \rightarrow n$. Adding anti-particles to both sides gives $\underline{p} \rightarrow n + \bar{\nu} + \beta^+$, where ν is the electron neutrino, $\bar{\nu}$ is its anti-neutrino and β^+ is an electron.

³ A caveat must be added for beta processes to allow for charge cancellation and reactions that change a proton to a neutron and vice versa.

the nucleons together to form a nucleus. *Binding energy* (equation 13.1-9) is the sum of the total proton mass, using the hydrogen: 1.007825 AMU, to correct for the electrons, with the total neutron mass: 1.008665 AMU, minus the atomic weight: $atwt(Z,N)$ in AMU multiplied by the conversion from AMU to MeV to express the results in energy units.

Table 13.1-3 compares the binding energy with related terms. *Mass Defect* is the mass difference that is caused by the binding forces. The mass defect per nucleon is called the *Packing Fraction*. The *binding energy* is the energy equivalent of the *mass defect* using $E = m*c^2$.

"Q" has the same meaning as in chemistry. In this case, it is the energy released in a nuclear reaction.

Table 13.1-3 Nuclear Binding	
Binding Energy	$BE = Z*m_H + N*m_n - m(A,Z)*c^2$ where $m(A,Z)$ is the isotopic mass.
Mass defect	$\delta = N*m_n + Z*m_H - A - m(A,Z)$
Packing fraction	$f = \delta/A$
Q	The total kinetic energy change in a nuclear reaction. Positive Q means an exothermic and negative means an endothermic reaction

Binding Energies of the Stable and Long Lived Nuclides

Computer Program 13-1 is simple, but tedious to input the 864 values of Z, A and $atwt(Z,A)$ that are obtained from the chart of the nuclides. It uses these data to calculate the binding energy vs A of the nuclei. The results are a fairly smooth curve with notable breaks corresponding to the strong binding of the alpha particle. This strong bind of the alpha lead to the suggestion that some nuclides have internal alpha particle groupings.

Program 13-1 Plotting the Binding Energy

DIM a(500), z(500), atwt(500), n(500)

DATA 1,1,1.007825,2,1.2,0.014,3,2,3.01603,4,2,4.00260,6,3,6.0152,7,3,7.016,9,4,9.01218,10,5,10.0129,11,5,11.00931,12,6,12.00,13,6,13.00335,14,7,14.00307,15,7,15.00011,16,8,15.99491,19,9,18.99840,20,10,19.99244,21,10,20.99395,22,10,21.99138,23,11,22.9898,24,12,23.98504,25,12,24.98584,26,12,25.98259,27,13,26.98153,28,14,27.97693,29,14,28.97649,30,14,29.97376,31,15,30.97376,32,16,31.97207,33,17,32.97146,34,16,33.96786,35,17,34.96885,37,17,36.965902,36,18,35.96755,38,18,37.96272,40,18,39.974,39,19,38.96371,40,19,39.964,41,19,40.96182,40,20,39.96259,42,20,41.95863,43,20,42.95878,44,20,43.96649,46,20,45.9537,48,20,47.9524,45,21,44.95592,46,23,45.95263,47,22,46.9518,49,22,48.94787,50,22,49.9448,50,23,49.9472,51,23,50.9440,50,24,49.9461,52,24,51.9405,53,24,52.9402,54,24,53.9389,55,25,54.9381,54,26,53.9566,56,26,55.9349,57,26,56.9354,58,26,57.9333,59,27,58.9332,58,28,57.9353,60,28,59.9496,61,28,60.9310,62,28,61.9283,63,29,62.9298,65,29,64.9278,64,30,63.9291,66,30,65.926,67,30,66.9485,68,30,67.9249,70,30,69.9253,69,31,68.9257,71,31,70.9249,70,32,69.9243,73,32,72.9234,74,32,73.9219,76,32,75.9214,75,33,74.9216,74,34,73.9225,76,34,75.9192,77,34,76.9199,78,34,77.9173,80,34,79.9165,82,34,81.9167,79,35,78.9183,81,35,80.9163,78,36,77.9204,80,36,79.9164,82,36,81.9135,83,36,82.9141,84,36,83.9115,86,36,85.9106,85,37,84.9117,87,37,86.9091,84,38,83.9134,86,38,85.9094,87,38,86.9089,88,38,87.9056,89,39,88.9054,90,40,89.9043,91,40,90.9053,92,40,91.9046,94,40,93.9061,96,40,95.9082,93,41,92.9069,92,42,91.9063,94,42,93.9047,95,42,94.9058,96,42,95.9046,97,42,96.9058,98,42,97.9055,100,42,99.9076,96,44,95.9076,98,44,97.9055,99,44,98.9061,100,44,99.9030,102,44,101.9037,104,44,103.9055,103,45,102.9048,102,46,101.9049,104,46,103.9036,105,46,104.9046,106,46,105.9032,108,46,107.9039,110,46,109.9052,107,47,106.9051,109,47,108.9047,106,48,105.907,108,48,107.904,110,48,109.903,111,48,110.904,112,48,111.903,113,48,112.9046,114,48,113.9036,116,48,115.905,113,49,112.904,115,49,114.9041,112,50,111.904,114,50,113.903,115,50,114.9035,116,50,115.9021,117,50,116.9031,118,50,117.9018,119,50,118.9034,122,50,121.9034,124,50,123.9052,121,51,120.9038,123,51,122.9041,120,52,119.9049,122,52,121.903,123,52,122.9042,124,52,123.9028,125,52,124.9044,126,52,125.9032,128,52,127.9047,130,52,129.9067,127,53,126.9004,124,54,123.9061,126,54,125.9042,128,54,127.9035,129,54,128.9048,130,54,129.9035,131,54,130.9051,132,54,131.9042,134,54,133.9054,136,54,135.9072,133,55,132.9051,130,56,129.9062,132,56,131.9057,134,56,133.9043,135,56,134.9056,136,56,135.9044,137,56,136.9056,138,56,137.905,138,57,137.9068,139,57,138.9061,136,58,135.907,138,58,137.9057,140,58,139.9053,142,58,141.909,141,59,140.9074,142,60,141.9075,143,60,142.9096,144,60,143.9099,145,60,144.9122,146,60,145.9127,148,60,147.9165,150,60,149.9207,144,62,143.9117,147,62,146.9146,148,92,147.9146,149,62,148.9196,150,62,149.917,152,62,151.9195,154,62,153.922,151,63,150.9196,153,63,152.9209,152,64,151.9195,154,64,153.9207,155,64,154.9226,156,64,155.9221,157,64,156.9339,158,64,157.9241,160,64,159.9271,156,66,155.9238,158,66,157.924,160,66,159.9248,161,66,160.9266,

162,66,161.9265,163,66,162.9284,164,55,163.9288,165,67,164.9033,162,68,161.9288,164,68,163.9293,166,68,165.9304,167,68,166.932,168,68,167.9324,170,68,169.9355,169,69,168.9344,168,70,167.9339,170,70,169.9349,171,70,170.9365,172,70,171.9366,173,70,172.9383,174,70,173.939,176,70,175.9427,175,71,174.9409,176,71,175.9426,174,72,173.9403,176,72,175.9416,177,72,176.9435,178,72,177.9439,179,72,178.9460,180,72,179.9468,180,73,179.9415,181,73,180.948,180,74,179.947,182,74,181.9483,184,74,183.951,186,74,185.9543,185,75,184.953,187,75,186.956,184,76,183.9526,186,76,185.9639,187,76,186.956,188,76,187.956,189,76,188.958,190,76,189.9586,192,76,191.9614,191,77,190.9609,193,77,192.9633,190,78,189.96,192,78,191.9614,194,78,193.9628,195,78,194.9648,196,78,195.965,198,78,197.9675,197,79,196.9666,196,80,195.9658,198,80,197.9668,199,80,198.9683,200,80,199.9683,201,80,200.9703,202,80,201.9706,204,80,203.9735,203,81,202.9723,205,81,204.9745,204,82,203.973,206,82,205.9745,207,82,206.9759,208,82,207.9766,209,83,208.9804,208,83,207.9813,209,84,208.9825,226,88,226.0254,227,89,227.0278,228,90,228.0278,229,90,229.0316,230,90,230.0331,232,90,232.0382,231,91,231.0359,232,92,232.0372,233,92,233.0395,234,92,234.0409,235,92,235.0439,236,92,236.0457,238,92,238.0506

```

1 SCREEN 11: VIEW (0, 0)-(639, 479)
2 WINDOW (-10, 10)-(250, -1)
3 i = 0: CLS
4 DO
5 i = i + 1
6 READ a(i)
7 READ z(i)
8 READ atwt(i)
9 n(i) = a(i) - z(i)
10 LOOP UNTIL a(i) = 238
11 iend = i
12 LINE (0, 0)-(240, 0):LINE (0, 0)-(0, 9)
13 FOR i = 20 TO 220 STEP 20
14 LINE (i, 0)-(i, .3)
15 NEXT i
16 FOR i = 1 TO 9
17 LINE (0, i)-(5, i)
18 NEXT i
19 FOR i = 1 TO iend
20 be = ((z(i) * 1.007825 + n(i) * 1.008665 -
21 atwt(i)) / a(i)) * 931.5
22 CIRCLE (a(i), be), .3
23 NEXT i
    END

```

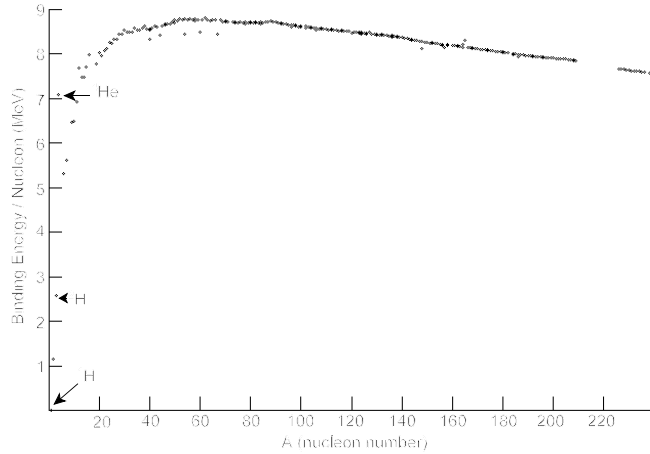
The program begins with A, Z, N, atwt being dimensioned to 500 to index these quantities. The data statement contains 288 sets of three data (864 total) for A, Z and atwt. (Note: the program that is provided contains all of the data on one line as required for BASIC). Lines 1 and 2 setup a window of 260x11. The index i is set to zero to use a Do-Until loop contained in lines 4-10. Lines 6-8 read A, Z, atwt data, three at a time. Line 9 calculates N. Line 11 records the last index used to read in the data. Line 12 draws the coordinates. Lines 13-18 draws the tick marks. Lines 16-23 plots the binding energy vs A. Lines 20-21 calculate the binding energy in MeV. Line 22 plots the points as small circles. Figure 13.1-6 shows the annotated results. The binding energy of helium is notable because of its very large binding energy. Hydrogen has a binding energy of zero since there is only one nucleon. Deuterium is pointed out because of its low binding energy. The curve is not as smooth as expected but the points were checked to assure

against typographical errors. The data includes the stable nuclides and heavy nuclides with lifetimes greater than 1E6 years. Notice the gap in the data where there are no stable nuclides nor nuclides with long half-lives. The light nuclei (Figure 13.1-7) show evidence of alpha particle sub-groupings

by increases in the binding for nuclei with A in multiples of 4.

The Binding Force is Proportional to A

Since Figure 13.1-6 is a plot of the *binding energy per nucleon* and it is fairly flat, the first observation is that the nuclear force: $f_n \propto A$. This contrasts with the Coulomb force that goes like $f_C \propto q_1 * q_2 / r$ i.e. the square of the charges, or the gravitation force that goes as $f_G \propto m_1 * m_2 / r$, i.e. the square of the masses. The constancy of the binding energy per nucleon suggests that the nuclear force has a very short-range that attracts only one nearest nucleon (like a coiled chain).



Nuclear Stability

Nuclear stability requires that a nuclear mass be less than the combined masses of the particles into which it could disintegrate. For example the following reaction cannot occur: $Li^7 \rightarrow He^4 + H^3$ because $7.016 < 4.0026 + 3.01605 = 7.01865$. Similarly $He^5 \rightarrow He^4 + n$ is possible. To show that this is true, the mass of He^5 can be found from the reaction $Li^7 + H^2 \rightarrow He^4 + He^5$. Knowing the masses of Li^7 , H^2 , and He^4 , and the kinetic energy carried by the reacting products, gives He^5 as having 5.0137 AMU, hence $5.0137 > 4.0026 + 1.008665 = 5.011265$.

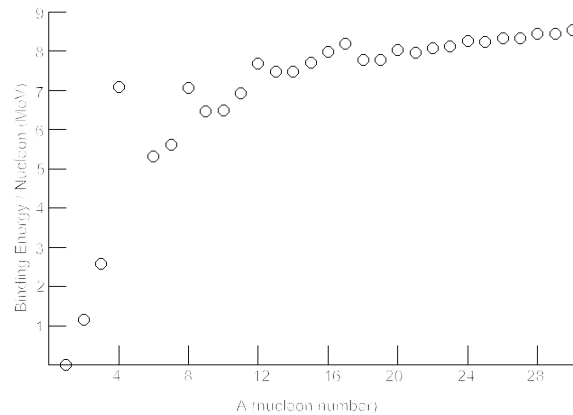


Fig. 13.1-7 Evidence of Alpha Particle Structure in Nuclei

13.2 Nuclear Models

The properties of nuclei from experiments may be tabulated, but while a tabulation may be useful for practical applications, it affords little understanding until nuclear models are constructed to systematize the knowledge. Since we have no experience in the nuclear realm, these models must be built from quantum mechanical models of atoms, and analogies from macroscopic physics such as the similarity of an atom to a liquid drop.

13.2.1 The Liquid Drop Model

The short-range of the nuclear force suggests a model analogous to a liquid. Symmetry

suggests that the nucleus should be spherical or nearly so. Energy given to the drop such as by the capture of a neutron may cause the shape of the drop to oscillate. In the extreme it may form a dumbbell and fission into two, and rarely, three particles. This model is analogous with the forces acting in a liquid drop with nuclear forces. To develop this model we first need a relationship for the size of the nuclear droplet to its mass.

Ways of Measuring the Radius of the Nucleus

The mass of a nucleus is related to A , the number of nucleons that it contains. If the nucleus is spherical with a well defined radius then A is related to the radius r as: $A(r) = ct * r_n^3$. In Section 12.9, we estimated the radius of Po^{210} to be 1.28 fm. Substituting this into $A(r)$ gives $ct = 4.6$ fm hence $r_n = 0.25 * A^{1/3}$ fm. This value for the nuclear radius was found by illustrating the use of the WKB approximation for calculating alpha particle decay. It is not the most accurate method for determining the nuclear size.

The first measurement of the nuclear radius was by Rutherford using alpha particle scattering from nuclei. Since then, the radius has been repeatedly measured for many isotopes using many alpha particle energies to find: $r_n = 1.3 + 1.35 * A^{1/3}$ fm. A similar method using protons scattered from nuclei, using a wide range of proton energies gives, gives: $r_n = 1.25 * A^{1/3}$ fm.

Another method using isobaric comparison i.e. comparing nuclei with the same mass number but having pp, and np groupings with the assumption that the nuclear force is charge independent gives $r_n = 1.67 * A^{1/3}$ fm.

Another method uses μ^- meson capture by a proton to form an atom with the mass $207 * m_e$ meson orbiting and penetrating the nucleus. When this is interpreted using Dirac's relativistic quantum mechanics the result give $r_n = 1.17 * A^{1/3}$ fm.

High energy electrons have probed the nuclear diameter under the assumption that protons are point charges. This method gives $r_N = (1.07 \text{ to } 1.15) * A^{1/3}$. For a working relationship between the nuclear radius and the nuclear mass we use equation 13.2-1.

$$r_N = 1.2 * A^{1/3} \text{ fm} \quad 13.2-1$$

13.2.2 Semi-Empirical Mass Formula

Mass of the Nucleons

Before extensive measurements of isotopic masses had been made, an empirical formula for estimating the masses was developed for their estimation. While a Taylor's series in Z , A and parity could have been developed, it is more constructive to use physical insight into the parts making up the mass. For this reason it is called the semi-empirical formula initially developed by Weissacker. The dominant term is the sum of the free masses of the protons and neutrons constituting the nucleus. However the Chart of the Nuclides provides the mass of a neutral atom i.e. an electron for each proton. Hence instead of the mass of the proton, the neutral hydrogen (H^1) mass is used to account for the electron mass. Thus the largest term in atomic mass units is: $1.007825 * Z + 1.008665 * (A-Z) = 1.008665 * A - 0.00084 * Z$.

Binding Energy per Nucleon

The sum of the masses of the free nucleons is reduced by the binding energy. Figure 13.1-6 shows that the binding energy per nucleon is fairly constant hence the correction: $C_1 = -c_1 * A$.

Surface Term

At the surface of the nucleus, the outward bonds do not connect. This requires a surface correction: $C_2 = c_2 * A^{2/3}$

Neutron-Proton Pairing

Studying the Chart of the Nuclides, shows that the stable nuclides tend to form neutron-proton pairs. Hence the correction $C_3 = c_3 * (A/2 - Z)^2 / A$ has a minimum if $N = Z$. Notice the energy is parabolic in neutrons or protons. The probability of the pairing is inversely proportional to the number of nucleons, hence the $1/A$ term.

Electrostatic Repulsion

A nucleus may be thought of as a sphere of charges. Electrostatic energy is: $E = 1/2 * C * V^2$. (9.1-34). The capacitance of a sphere is $C = r / \kappa$ (equation 9.1-32). The voltage from a charge is $V = \kappa * q / r$. This is like Newton's gravity problem hence the voltage on the surface of the nucleus from all of the protons in the sphere is $V = \kappa * Z * e / r$, and $E = 1/2 * \kappa * Z^2 * e^2 / r$ (Joules). (An exact calculation replaces $1/2$ with $3/5$). Using $r = 1.2 * A^{1/3}$ fm for the radius of the nucleus with $\kappa = 1 / (4 * \pi * \epsilon_0) = 9.9876E9$. Converting to MeV and to AMUs, we find $E = 3/5 * \kappa * Z^2 * e / (1.2E-15 * A^{1/3} * 931.5E6) = 8.59E-4 * Z^2 / A^{1/3}$ hence $c_4 = 8.59 E-4$ and the electrostatic terms is $C_4 = c_4 * Z^2 / A^{1/3}$.

N, Z Even-Odd Term

Let $P(x)$ be a vector of magnitude $1/2$ that points up when x is odd and points down i.e. it is $-1/2$, when x is even. Referring to the Chart of the Nuclides, the most stable nuclides have N and Z even. Less stable nuclides have N even when Z is odd or Z even when N is odd. The least stable nuclides have N and Z odd. A term that expresses this is $C_5 = c_5 * [P(N) + P(Z)] / A^{3/4}$ where $A^{3/4}$, as far as I can tell, is an empirical relationship. Notice that the condition that Z or N are odd, but not both, is the same as A being odd.. By fitting nuclear mass data we find $c_5 = 0.036$. Thus the parity term is: $C_5 = 0.036 * [P(N) + P(Z)] / A^{3/4}$

Semi Empirical Mass Formula

Equation 13.2-2 summarizes these results.

$$m = 1.008665 * A - 0.00084 * Z - a_1 * A + a_2 * A^{2/3} + a_3 * (A/2 - Z)^2 / A + 8.58E-4 * Z^2 / A^{1/3} - a_5 * [P(N) + P(Z)] / A^{3/4} \quad 13.2-2$$

To evaluate these constants, we restrict the consideration to the stable nuclides with A

$$\partial m / \partial Z = 0 = -0.00084 - 2 * a_3 * (A/2 - Z) / A + 1.716E-3 * Z / A^{1/3} \quad 13.2-3$$

$$Z = \frac{A}{1.746 + 0.0138 * A^{2/3}} \quad 13.2-4$$

odd so the fifth term vanishes. A plot of mass vs Z shows a parabola-like shape that is minimum for the stable nuclides, a condition given by $\partial m / \partial Z = 0$ (equation 13.2-3). The value of a_3 may be found from the Chart of the Nuclides, for example, $^{93}\text{Nb}^{41}$ with 52 neutrons has mass 92.906 in one stable isotope. This gives $a_3 = 0.124$. Substituting this into 13.2-3 gives equation 13.2-4 which shows for $A = 1$, the ratio $A/Z = 2$ as it is for light nuclei.

Now we need to solve for the constants a_1 and a_2 using other nuclides. The mathematics is tedious and prone to error therefore we will find these constants using program 13-2.

Program 13-2 Find Constants for the Empirical Mass Formula

```

1 DIM ct(2), a1ct(2), a2ct(2)
2 cA = 1.008665: cZ = .00084: a3 = .124: a4 =
3 .000858: CLS
4 FOR i = 1 TO 2
5 IF i = 1 THEN
6 m = 9.01218: A = 9 'Beryllium-9
7 ELSE
8 m = 126.9004: A = 127 'Iodine-127
9 END IF
10 Z = A / (1.746 + .01375 * A ^ .667)
11 ct(i) = -m + cA * A - cZ * Z + a3 * (A / 2 - Z)
12 ^ 2 / A + a4 * Z ^ 2 / A ^ .333
13 a1ct(i) = A: a2ct(i) = A ^ .667
14 'PRINT at(i), a1(i), a2(i)
15 NEXT i
16 a1 = (a2ct(2) * ct(1) - a2ct(1) * ct(2)) /
17 (a2ct(2) * a1ct(1) - a2ct(1) * a1ct(2))
18 a2 = (a1 * a1ct(1) - ct(1)) / a2ct(1)
19 PRINT a1, a2

```

Equation 13.2-2, using 13.2-3, is a function of A only. The coefficients a_1 and a_2 can be found for two different masses having odd A , to make the parity term vanish. The constants are combined as $ct(i)$ and the coefficients of a_1 and a_2 are $ct1(i)$ and $ct2(i)$, respectively. where i is the index for the two masses. These are dimensioned in line 1. The constants used in equation 13.2-2 are defined in lines 2-3. The For-Next loop from 4-15 performs the calculation. The If statement from 5-9 associates the atomic weight and A for beryllium with the index $i=1$, and $i=2$ for iodine. Lines 10-15 relate A to Z via equation 13.1-15. Line 13 calculates $ct(i)$ and line 13 calculates $c1(i)$ and $c2(i)$. Line 16 solves for a_1 by elimination. Line 18 calculates a_2 by substitution. The result is $a_1 = 1.8104E-2$ and $a_2 = 2.028E-2$.

Program 13-3 Weisacker Mass Formula

Rather than directly using equation 13.2-2, it is convenient to be able to calculate the mass of an isotope by specifying its chemical symbol and atomic mass which is what this program does.

```

1 DIM nam$(105)
2 DATA H,He,Li,Be,b,C,N,O,F,Ne,Na,Mg,Al,Si,P,S,Cl,Ar, K,Ca,Sc,Ti,V,Cr,Mn,Fe,Co,Ni,Cu,Zn,
3 Ga,Ge,As,Se,Br,Kr,Rb,Sr,Y,Zr,Nb,Mo,Tc,Ru,Rh,Pd,Ag,Cd,In,Sn,Sb,Te
4 DATA I,Xe,Cs,Ba,La,Ce,Pr,Nd,Pm,Sm,Eu,Gd,Tb,Dy,Ho,Er,Tm,Yb,Lu,Hf,Ta,W,Re,Os,Ir,Pt,Au,Hg,
5 Th,Pb,Bi,Pa,At,Rn,Fr,Ra,Ac,Th,Pa,U,Np,Pu,Am,Cm,Bk,Cf,Es,Fm,Md,No,Lr,Rf:CLS
6 FOR Z = 1 TO 52
7 READ nam$(Z)
8 NEXT Z
9 FOR Z = 53 TO 104
10 READ nam$(Z)

```

Line 1 dimensions the problem for 105 elements. Lines 2-5 lists the chemical names from hydrogen to rutherfordium. Lines 6-8 read in the first 51 elements and lines 9-11 do the same for the remaining isotopes. Lines

```

11 NEXT Z
12 INPUT "Chemical Name (First letter
13 capitalized) ", cnam$
14 INPUT "Input Mass ", A
15 Z = 0
16 DO
17 Z = Z + 1
18 LOOP UNTIL nam$(Z) = cnam$
19 N = A - Z
20 cA = 1.008665: cZ = .00084: a1 = .0181
21 a2 = .0203: a3 = .124: a4 = .000858
22 a5 = .035
23 IF A MOD 2 = 0 THEN
24 parity = 0
25 END IF
26 IF Z MOD 2 <> 0 AND N MOD 2 <> 0
27 THEN
28 parity = 1
29 END IF
30 IF Z MOD 2 = 0 AND N MOD 2 = 0 THEN
31 parity = -1
32 END IF
33 benergy = cA * A - cZ * Z - a1 * A + a2 * A
34 ^ (2 / 3) + a3 * (A / 2 - Z) ^ 2 / A + a4 * Z ^ 2
35 / A ^ (1 / 3) + a5 * parity / A ^ (3 / 4)
36 PRINT benergy; "AMU": END

```

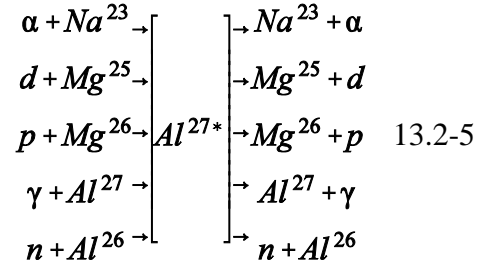
12-13 instruct you to input the chemical symbol which is stored as cnam. Line 14 instructs you to input the atomic mass as A. Line 15 initializes Z for a Do loop to scan all of the symbols until it finds the one you input to find the proton number. Line 19 calculates the neutron number. Lines 20-22 provide constants for equation 13.1-13. Lines 23-29 determine the parity of A and N to give 1 if N and Z are both odd, -1 if they are both even and zero if they have opposite parity. This multiplies constant a5. Equation 13.2-2 is calculated in lines 33-35 and printed by line 36. For example for uranium ²³⁸U it calculates 238.0079. The Rubber Handbook gives 238.0313.

13.2.3 The Compound Nuclear Model

In Section 13.1.2, we used the chart of the nuclides to identify different reactions that may occur when a particle is incident on a nucleus. If it is above the nuclear binding energy (> 8 MeV) the reaction occurs between the incident particle and a nucleon - as if there were no nucleus. Below the binding energy, the incident particle becomes part of a compound nucleus that is formed of the initial target nucleus and the incident particle. This is meaningful only if the compound nucleus exists for a time longer than the time required for the projectile to transient the target. For example, it takes a thermal neutron 1E-19 s to transient a Hg²⁰⁹ nucleus. Hence a meaningful compound nucleus model must last longer than this. If the energy of the projectile is about the same as the nucleons in the target, it quickly loses its identity and shares energy with the other nucleons to form a compound nucleus in an excited state. This excited state continues until the compound nucleus reconfigures to greater stability - perhaps by particle and energy emission.

De-excitation may occur by gamma emission in which case the *residual nucleus* (the nucleus that results after de-excitation) retains the *A* and *Z* of the compound nucleus but loses excitation energy. It may de-excite by emitting nucleons with changes in the *A* and *Z* of the compound nucleus to form the residual nucleus with the excitation energy being emitted in the form of the kinetic energy of the ejected particles. Certain compound nuclei may fission into two massive particles with emission of neutrons, neutrinos, gammas and betas (see Table 11.13-1).

The long life causes the compound nucleus to "forget" how it was formed, therefore how it decays does not depend on how it was formed. Equation 13.2-5 illustrates this with five reactions that may form the same excited compound nucleus (denoted by the asterisk *). If the compound nucleus decays into a nucleus that is the same as the initial nucleus it is called a scattering reaction. If energy is conserved, it is called *elastic scattering*, if it is not conserved, it is called *inelastic scattering*. In addition to these non-transforming reactions, the compound nucleus may decay by any of the five reactions shown. The likelihood of each of these reactions is expressed as the *branching ratio* specific to a particular reaction.



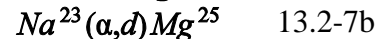
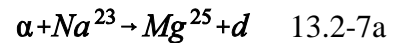
Reaction Energy

Mass-energy is conserved in nuclear reactions. The excitation energy is the sum of the binding energy of the incident particle given to the compound nucleus plus whatever kinetic energy in the center of mass system that the particle carried. This is shown in equation 13.2-6 in which the reduced mass is used to convert the energy of the incident particle (E_{ilab}) in the laboratory system to its amount in the laboratory system where the target particle is at rest. The binding energy (E_{bind}) is the amount of energy that is required to remove the particle from the compound nucleus.

$$E_{excit} = A * E_{ilab} / (A + a) + E_{bind} \quad 13.2-6$$

Simplified Reaction Notation

We have been writing a nuclear reaction similarly to chemical reactions i.e., like equation 13.2-7a, for example. A shorter way of writing the reaction is shown as equation 13.2-7b. Here, the incident and exit particles are shown in parenthesis, preceded by the initial nucleus and followed by the final residual nucleus. These notations do not show explicitly the compound nucleus which, in this case is the stable nucleus Na^{20} .



Energy Balance

Consider the reaction X(x,y)Y where the capital letters stand for nuclei and the lower case letters for incident and exiting particles. The mass energy balance for the reaction is given by

equation 13.2-8, Where the E_s are the kinetic energies of the identified particles and the M_s are the mass-energies of the identified particles ($m \cdot c^2$). The quantity Q was defined in Table 13.1-3 as the final kinetic energy minus the initial kinetic energy (equation 13.2-9). Using this with equation 13.2-8 shows that Q is also the difference between final mass-energy and initial mass-energy.

$$(E_x + M_x) + (M_Y) = (E_y + M_y) + (E_Y + M_Y) \quad 13.2-8$$

$$Q = E_y + E_Y - E_x \quad 13.2-9$$

$$Q = M_y + M_Y - M_x - M_X \quad 13.2-10$$

$$Q = E_y \cdot (1 + m_y/M_Y) - E_x \cdot (1 + m_x/M_X) - 2/M_Y \cdot \sqrt{(E_x \cdot E_y \cdot m_x \cdot m_y)} \cdot \cos(\theta) \quad 13.2-11$$

The Q of a reaction may be measured by measuring each of the quantities on the right side of equation 13.2-9, but it is difficult to measure the recoil energy of the residual nucleus (E_Y). Using momentum balance, the recoil energy can be replaced and equation 13.2-11 is obtained.

Energy Conservation

If a particle is incident on a nucleon or if particles exit from a nucleus, these energies must also be considered (equation 13.2-12), where Q is the reaction energy which is positive if exothermic and negative if endothermic. If one or more particles exit, their energies must be included as well as the recoil energy of the nucleus.

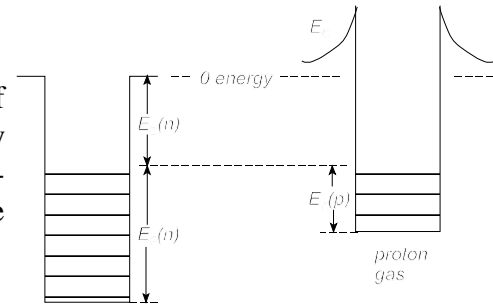
$$T_{CM}(incident) = T_{CM}(exiting) + Q \quad 13.2-12$$

Example of Energy Balance

Consider the reaction: $Li^7 + H^1 \rightarrow 2 \cdot He^4$. The mass difference is $7.01600 + 1.007825 - 2 \cdot 4.0026 = 0.018625 \cdot 931.5016 = 17.34 \text{ MeV } Q$.

Center-of-Mass Collision Energy

Section 11.13.2, addressed the elastic scattering of neutrons from nuclides. Here, the concern is for the energy of impact. The velocity of the projectile (n) in center-of-mass system (CM) is: $v_{nC} = A \cdot v_{nL} / (A + m_n)$, where A is the target mass. The velocity of the target $v_{AC} = m_n \cdot v_{nL} / (A + m_n)$. Since the target is not moving in the laboratory system, the energy of the projectile equals the energies of the projectile and target in the CM: $T_{CM} = \frac{1}{2} \cdot m_n \cdot v_{nL}^2 = \frac{1}{2} \cdot m_n \cdot v_{nC}^2 + \frac{1}{2} \cdot A \cdot v_{AC}^2 = \frac{1}{2} \cdot m_n \cdot [A \cdot v_{nL} / (A + m_n)]^2 + \frac{1}{2} \cdot A \cdot [m_n \cdot v_{nL} / (A + m_n)]^2$. Rearranging gives equation 13.2-13, where μ is the reduced mass: $1/\mu = 1/m_n + 1/A$. (Like resistors.) Notice as $A \rightarrow \infty \mu \rightarrow m_n$ and $v_{nC} \rightarrow v_{nL}$.



$T_{CM} = \frac{1}{2} \cdot v_{nL}^2 \cdot m_n \cdot A / (m_n + A)$
 Fig. 13.2-13 Square Well Potentials for Neutrons and Protons (note the Coulomb energy)

Nuclear Shape

Nuclides show electric and magnetic moments; these moments arise from asymmetries in the nuclear shape. Nuclear sphericity only occurs at the shell closings when A or Z have the magic

numbers: 8, 16, 28, 38, 50, 82. and 126.

Fermi Gas

A nucleus containing fermions may be considered in the context of a Fermi gas model consisting of non-interacting nucleons freely moving about in a spherical potential well having the nuclear diameter. The well depth is adjusted so that the Fermi energy raises the highest nucleons to an energy corresponding to the observed binding energies (Figure 13.2-1).

Protons and neutrons fill the well in accord with the Pauli principle that allows two particles of a type, one with spin up and the other with spin down to fill each cell in phase space having volume $(2\pi\hbar)^3$. The Fermi energy of the neutrons or protons is given by equation 13.2-14, where p_F is the momentum. For a spherical well of radius $r = 1.2 fm$, equation 13.2-15 is found where n is the number of protons (Z) or neutrons ($A-Z$).

$$E_F = p_F^2 / (2 * m) \quad 13.2-14$$

$$E_F = 54(n/A)^{2/3} \quad 13.2-15$$

The well depth is set to set the highest neutron energy state equal to the observed binding energy - about 8 MeV. If the A of the nucleus is fairly large, $N > Z$ and the well depths for the protons and neutrons must differ because for the over-all stability of the nucleus, the energy of the highest energy proton state and the highest energy neutron state must be equal (except for the proton-neutron mass difference). However $E_B \sim n^{2/3}$ and must be smaller because of the fewer protons than neutrons. The depth of the proton well is greater than $E_F(p)$ only by the binding energy of the last proton (6-8 MeV). Thus the proton well depth is considerably less than the neutron well depth because neutrons are attracted by the nuclear force while protons are attracted by the nuclear force but repelled by the Coulomb force.

For example, for PB^{208} , $Z = 82$, $N = 126$, $EB \approx 6$ MeV, and the bottom of the neutron well lies 44 MeV below zero energy and the bottom of the proton well lies at 34 below zero.

This shows the importance of the zero-point energy in nuclear matter and the typical high velocities of nucleons in nuclei. The model suggest that nucleon collisions will not often transfer small amounts of momentum to the nucleus because the nucleon momentum states near the origin are filled. Much momentum must be transferred or the collision cannot occur as shown by nuclear reactions.

The End

Alas dear reader, I must end. I am 79 and had planned this current chapter to extend to much more detailed discussion of nuclear models and elementary particles and additional chapter on Solid State Physics but my time is running out. I don't know if I will live long enough for this completion and decided to give you what I have done. I must say it has been a pleasure to organize what I studied over the years, to give you tricks that I learned. My work has been in experimental nuclear physics and reactor physics but putting physics together to see the global extent has given me great satisfaction. I think it is one of the magnificent intellectual pursuits of man.